APPLICATION OF EVOLUTIONARY ALGORITHMS FOR THE MODEL-FREE DETERMINATION OF SCATTERING MATRIX AND THE NUMERICAL DIAGONALIZATION OF NILSSON-MODEL HAMILTONIAN

A.N. Vodin[1], V.Yu. Korda[2], A.N. Dovbnya[1], A.S.Molev[2], L.P. Korda[1]

[1]National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine
e-mail: vodin@kipt.kharkov.ua

[2]Scientific and Technological Centre of Electrophysics Ukrainian National Academy of Sciences, Kharkov, Ukraine
e-mail: v_yu_korda@yahoo.com

A new model-free approach for analyzing the intermediate-energy light nucleus-nucleus elastic-scattering differential cross-sections is proposed to extract the numerical dependencies of S-matrix modulus and nuclear phase on angular momentum directly from the experimental data via the evolutionary algorithm. The refractive and absorptive properties of the ^{16}O–^{16}O-interaction at E = 22…44 MeV/nucleon are studied. The new approach based on a genetic algorithm is presented to diagonalize the Hamiltonian matrix of the Nilsson-model for the deformed axially symmetric nucleus. The tests witness that the larger the matrix size the more effective the genetic approach becomes comparing to the traditional diagonalization methods.

PACS: 24.10.Ht, 02.60.Pn, 25.40. Lw, 23.20.-g

1. INTRODUCTION

Experimental and theoretical investigations of differential cross sections of the scattering of light nuclei by nuclei at $E \geq 15…20$ MeV/nucleon are still in focus for the analysis of such cross sections comprising clearly pronounced refractive structures allows to obtain valuable information on inter-nucleus interaction on small distances (see, e.g., [1–3]). Usually the differential cross sections of intermediate energy nuclei-nuclei elastic scattering are analyzed with help of complex optical potentials or scattering matrices having particular forms and a few number of free parameters the values of which are determined from the fitting of available experimental data. Unfortunately there are great variety of models leading in several crucial cases to different physical interpretations of the observed features of differential cross sections. Thus, it feels actual to pose the problem of extraction of the optical potential or the scattering matrix directly from the experimental data with making as less of model assumptions as possible. In the course, the quality of fitting the data can be improved if one chooses more flexible forms of optical potential or scattering matrix than those of conventional ones. With this in mind various initial forms were used to be simple parameterizations or found from the microscopic calculations further modified with additional terms being the full set function expansions (see, [4–6]). The real part of optical potential can also be chosen in the form being more general than the common Saxon-Woods or folding ones, allowing the non-monotonic behavior. For instance, in [7] the spline form potential assisted the folding one. These approaches turned out to be quit successful in analyzing the experimental data but should be treated as model dependent for they used *a priory* model visions for the optical potential or the scattering matrix.

In this paper we expose the novel model-free approach in which the numerical dependencies of the S–matrix modulus and nuclear phase shift on the angular momentum are extracted directly from the experimental data with help of the data fitting procedure using evolutionary algorithm with smooth deformations of these dependencies. On the basis of the proposed approach the refractive and absorptive properties of the ^{16}O–^{16}O-interaction at E = 22…44 MeV/nucleon are studied. Besides, we present the novel approach based on the use of a genetic algorithm which enables the numerical diagonalization of an arbitrary Hermitian matrix, with asymptotically exponential convergence. With help of the approach developed we solve the full problem on eigenfunctions and eigenvalues of the Hamiltonian matrix of Nilsson's model describing the one-particle states of the deformed nuclei, in which the nucleons move in the oscillator potential with axial symmetry.

2. EVOLUTIONARY ALGORITHM FOR SCATTERING MATRIX DETERMINATION

The scattering matrix in the angular momentum representation can be presented in the form

$$S_l = \eta_l \exp(2i\delta_l + 2i\sigma_l), \quad (1)$$

where η_l is the scattering matrix modulus, δ_l and σ_l are the nuclear and Coulomb phase shifts correspondingly.

We have developed the approach with help of which the modulus η_l and the nuclear phase δ_l as functions of angular momentum l are extracted directly from the experimental data (the Coulomb phase σ_l is assumed to poses the quasiclassic form of the point charge scattering by the uniformly charged sphere with radius R_C [8], which is legitimated by the high enough energy of scattering) without assuming any additional model assumptions. In our approach the values of η_l and δ_l for each l are treated as independent fitting parameters. Thus the parameter space of the variation problem has high dimension and one has to chose an adequate optimization method.

Evolutionary algorithm is the one of choice the application of which turned out to be quite efficient in solving such different and hard problems as the integration of differential equations [9], Thomson's problem on the minimum energy configuration of N point charges on a unit sphere [10], the optimization of thermodynamic analysis of phase transitions in ferroelectrics [11], the construction of the light exotic nuclei bound state wavefunctions [12, 13], the optimization of parameters of the energy dependent optical potential for the elastic scattering of heavy ions by nuclei [14], the analysis of spectra of quantum systems [15], the optimization of kinematics of nuclear physics experiments [16], etc.

The only condition the modulus η_l and the phase δ_l, must obey is the requirement for them to have smooth form. Alongside with that one should note that at $E \geq 15\ldots20$ MeV/nucleon the number of fitting parameters usually exceeds the number of experimental points. To avoid overfitting we have devised the special mutation operator with dispersion. The developed algorithm consists of the following steps:

1. The initial population of n individuals is created, each of which including the pair of real-valued vectors (η_l, δ_l) of dimension l_{max} with $l = 0,\ldots, l_{max}$ -1. The initial dependencies η_l and δ_l are chosen in the analytical form

$$\eta_l = \exp\left(-\mu_a g(L, L_a, \Delta_a)\right), \; 2\delta_l = \mu_r \exp\left(-\frac{L^2}{\Delta_r^2}\right),$$

$$g(L, L_a, \Delta_a) = \left[1 + \exp\left(\frac{L - L_a}{\Delta_a}\right)\right]^{-1}, \quad (2)$$

where $L = l + 1/2$, the parameter μ_a determines the magnitude of absorption, the parameters L_a and Δ_a (μ_r and Δ_r) characterize the size of the region of absorption (nuclear refraction) in the angular momentum space. These shapes of the modulus η_l and the nuclear phase δ_l meet the condition of smooth alteration with the increase of l (see [17]) and correctly account for the character of absorption and refraction in the process of scattering of light nuclei by nuclei at intermediate energies. The values of the parameters μ_a, μ_r, L_a, Δ_a and Δ_r from Eq. (2) are the random ones uniformly distributed in wide range. Thus this choice of the initial representation of η_l and δ_l does not influence the final result obtained with the algorithm at hand.

2. The elastic scattering differential cross section is calculated and the value of χ^2 is found for each of the individual from the population. The procedure uses the amplitude of elastic scattering in the form of the Legendre polynomials expansion. The differential cross section is determined by the squared modulus of this amplitude.

3. According to the χ^2 magnitude, two parent individuals are selected randomly from the population (the lower the magnitude of χ^2 the higher the probability to be selected) to create two offsprings using the transformation

$$\ln(\eta_l') = \ln(\eta_l)\left[1 + \frac{0{,}25aN(0,1)}{1 + \left(\frac{l - l_0}{d}\right)^2}\right],$$

$$\delta_l' = \delta_l\left[1 + \frac{0{,}25aN(0,1)}{1 + \left(\frac{l - l_0}{d}\right)^2}\right], \quad (3)$$

where a is the mutation amplitude, $N(0, 1)$ is the normally distributed one-dimensional random variable with zero mean and one standard deviation, l_0 is the random point of mutation, d is the dispersion characterizing the size of the mutation region.

The values of l_0 and $N(0, 1)$ are generated ones for each string of η и δ. The control parameters a and d alter in intervals $[a_{min}, a_{max}]$ and $[d_{min}, d_{max}]$ chosen by the user. These parameters are automatically tuned by the computational program in the specified intervals. The mutation operator (3) is nonlocal and if the values of a and d are set properly the transformed functions η_l' and δ_l' will be smooth. In order to provide monotonicity of the extracted functions η_l and δ_l we first find the minimum value χ^2_{mon} at which these functions still poses the property. Then while χ^2 decreases below χ^2_{mon} we apply the additional condition due to which the maximal relative alteration of the functions η_l and δ_l can not exceed some small value (say 10^{-4}, as in our case).

4. For each of two offsprings the elastic scattering differential cross section is calculated and χ^2 magnitude is determined.

5. According to the χ^2 magnitude, two individuals are selected randomly from the population (the higher the magnitude of χ^2 the higher the probability to be selected) to be substituted by the offsprings.

6. Go to step 3.

For the details of the selection procedure see exposition in [18].

3. RESULTS AND DISCUSSION

We have carried out the model-free study of the refractive and absorptive properties of ^{16}O–^{16}O-interaction at E = 22…44 MeV/nucleon. The choice of the ^{16}O + ^{16}O system is conditioned by the fact that there exit valuable discrepancies between the S-matrix module and deflection functions, characterizing absorptive and refractive properties of interaction, respectively, obtained with use of different models (phenomenological optical model, folding model and S-matrix model with Regge poles) (see [19, 20]). Therefore, it is important to get new reliable information on the scattering matrix in the cases under study.

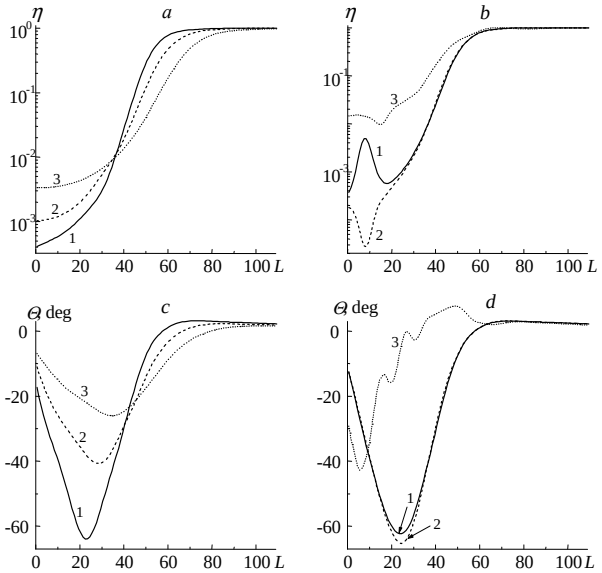

Fig. 1. *The modulus of scattering matrix (a, b) and the deflection function (c, d) for elastic $^{16}O - ^{16}O$-scattering: b, d and curves 1 (a, c) are for E = 350 MeV while curves 2 (a, c) are for E = 480 MeV and curves 3 (a, c) are for E = 704 MeV*

The results of analysis of differential cross sections of elastic ^{16}O–^{16}O-scattering at E = 350, 480 and 704 MeV carried out with help of the proposed approach are exposed in Figs. 1–3. The values of control parameters of the evolutionary algorithm are presented in Table alongside the χ^2 magnitudes and the total reaction cross sections σ_r. Besides the boundary values for a and d, the Table contains their initial (a_i and d_i) and final (a_f and d_f) values. The calculations utilized n = 100 and $R_C = 0{,}95 \cdot 2 \cdot 16^{1/3}$ [23]. The values of parameters of the representations (2) altered in wide intervals: L_a = 30,0…55,0; Δ_a = 4,0…14,0; Δ_r = 20,0…50,0; μ_a = 4,0…10,0, μ_r = 5,0…50,0 which provided for the various enough initial conditions. The amount of χ^2 calculations were $1{,}4 \cdot 10^6$, $3{,}8 \cdot 10^6$ and $2{,}0 \cdot 10^6$ iterations for E = 350, 480 and 704 MeV respectively.

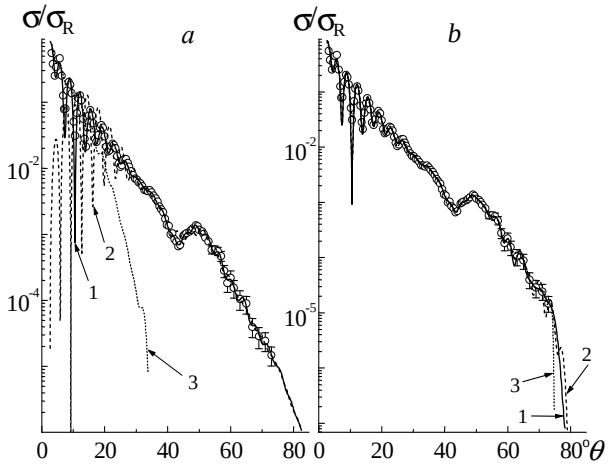

Fig. 2. *The ratio of the differential cross section of elastic $^{16}O - ^{16}O$-scattering at E = 350 MeV and its components to the Rutherford one: curves 1 – 3 (a) are the cross section and its refractive and absorptive components calculated with the S-matrix to which the curves 1 in Fig. 1, a, c correspond; curves 1 – 3 (b) are the cross sections calculated with the S-matrix to which the curves 1 – 3 in Fig. 1, b, d correspond. The points are the experimental data taken from [21, 22]*

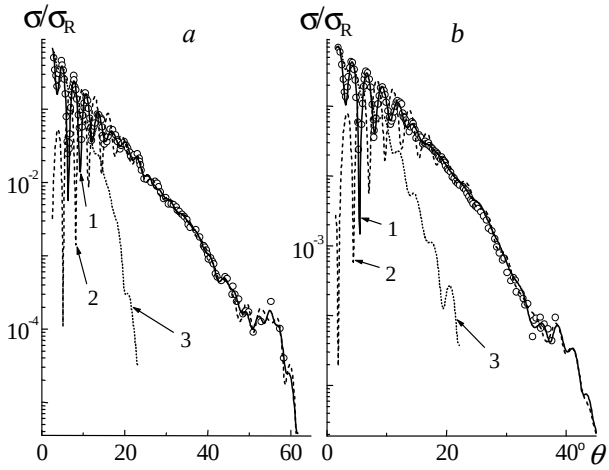

Fig. 3. *The same as in Fig 2, a but for E = 480 MeV (a) and E = 704 MeV (b). a, b show the results of calculations with the S-matrix to which the curves 2 and 3 in Fig. 1, a, c correspond. The points are the experimental data taken from [19]*

The calculated differential cross sections were symmetrized for the scattering of the identical nuclei. Figs. 2 and 3 show that the application of the developed approach allows to obtain good agreement between the calculated and measured cross sections. As in [19] the fitting of differential cross sections was held with the standard experimental error of 10%. The scattering matrix (1) for the cases shown on Figs. 1, *a*, *c* is characterized by smooth dependence on L since the extracted from the data module $\eta(L)$ and nuclear phases $\delta(L)$ are smooth monotonic functions of L. The boundary momenta of strong absorption L_{sa} defined from the correlation $\eta^2(L_{sa}) = 0{,}5$ with help of the dependencies $\eta(L)$ shown on Fig. 1, *a* acquire the values: L_{sa} = 58,1; 66,3 and 76,9 at E = 350, 480 и 704 MeV respectively. The deflection functions $\Theta(L) = 2(\delta_L - \delta_{L-1}) + 2d\sigma(L)/dL$ in

the cases under study have the forms typical of the cases of nuclear rainbow (see Fig. 1, *c*). The angles of nuclear rainbow corresponding to the minima of $\Theta(L)$ are equal to $\theta_R = 64$; 41 and 26° at E = 350, 480 и 704 MeV respectively. Note that the values of θ_R found at E = 350 and 480 MeV are in good agreement with the ones obtained on the basis of the folding model with the density dependent nucleon-nucleon interaction [19] and the nine-parameter S-matrix model [24].

Table. *The control parameters of the evolutionary algorithm used in the calculations of the differential cross sections of elastic $^{16}O - ^{16}O$-scattering at different energies*

Parameter	*Value*		
	350 MeV	*480 MeV*	*704 MeV*
l_{max}	120	135	160
a_{min}	10^{-2}	10^{-3}	10^{-5}
a_{max}	0,9	0,9	0,9
a_i	0,9	0,9	0,9
a_f	$1{,}3\cdot10^{-2}$	$1{,}2\cdot10^{-2}$	$1{,}6\cdot10^{-4}$
d_{min}	5,5	7,0	9,5
d_{max}	100,0	100,0	100,0
d_i	80,0	90,0	100,0
d_f	6,5	7,0	11,5
χ^2	2,5	1,9	1,1
σ_r, mb	1693	1581	1483

In order to clarify the degree to which the nuclear refraction and the strong absorption are responsible for the formation of different features of the elastic ^{16}O–^{16}O-scattering in different regions of scattering angles θ Figs. 2, *a* and 3 display the refractive and diffractive components of the cross sections studied calculated due to the method proposed in [25]. Figs. show that at $\theta \geq 20\ldots25°$ the refractive components (curves 2) almost totally replicate the respective differential cross sections, particularly they describe fine details of the rainbow "tail" in the cases of the scattering at E = 480 and 704 MeV. The diffractive components (curves 3) dominate in the regions of Fraunhofer oscillations at small angles θ.

When the values of dispersion d_{min} decrease to 2,0…2,5 and the procedure of additional control of monotonicity of the extracted dependencies $\eta(L)$ and $\delta(L)$ is switched off the behavior of the modulus $\eta(L)$ in the region of small momenta becomes non-monotonic. In this region there arise isolated structures ("humps") like the ones shown on Figs. 1, *b* (curves 1, 2) for the scattering at E = 350 MeV while the deflection function has the similar form as in Fig. 1, *c*. Note the values of those parameters of calculations for the curves 1, 2 in Fig. 2, *b* which differ from the ones mentioned in the table: $d_{min} = d_f = 2{,}0$, $a_{min} = 0{,}0$, $a_f = 6{,}4\cdot10^{-11}$, $\sigma_r = 1667$ mb (curve 1), $d_{min} = 2{,}0$, $d_f = 2{,}6$, $a_{min} = 10^{-3}$, $a_f = 6{,}5\cdot10^{-3}$, $\sigma_r = 1678$ mb (curve 2).

Fig. 2 shows that the use of the different forms of the scattering matrix to which the curves 1 and 2 in Fig. 1, *b, c* and the curves 1 in Fig. 1, *a, c* correspond gives equivalent quality of fit of the data although in the region of large enough scattering angles ($\theta > 70°$) some discrepancy between the differential cross sections obtained is however observed. Therefore, the detailed structure of $\eta(L)$ in the region of small momenta containing small "humps" the character size of which $\lambda_h \approx 7/k \approx 0{,}9$ fm (k is the wave number) is compared with the wave length of the relative motion of colliding nuclei $\lambda = 2\pi/k \approx 0{,}8$ fm should not be assigned any physical meaning and its existence is not necessary for the correct explanation of the available experimental data. Thus the monotonic function (see Fig. 1, *a*) smoothly altering from small values to unity with the increase of L may be used as a modulus $\eta(L)$ of scattering matrix for the $^{16}O + ^{16}O$ system at the energy under study.

Note that when the program of calculations automatically selected the conditions for $S(L)$ corresponding to the case of week nuclear refraction we found the differential cross section (curve 3 in Fig. 2, *b*) for which $\chi^2 = 0{,}6$ ($d_f = 9{,}3$, $a_{min} = 10^{-3}$, $a_f = 9{,}2\cdot10^{-3}$, $\sigma_r = 1590$ mb). In the context the question remains on how deep physical meaning might have the oscillation structures in the modulus of the scattering matrix and the deflection function (see curves 3 in Fig. 1, *b, c*). Obviously this result should be considered as a physical nonsense and only the smooth behavior of $\eta(L)$ and $\delta(L)$ without any additional structures characterizes correctly the effects of absorption and refraction in the ^{16}O–^{16}O-scattering at the energies under study.

These conclusions contradict the results of the work [20] in which the differential cross section of elastic ^{16}O–^{16}O-scattering was analyzed on the basis of the ten-parameter S-matrix model allowing for the pole factor and the strong influence of Regge poles and zeros of S-matrix lying in the vicinity of the real axes in the complex L plane on the refractive behavior of the studied cross sections at E ≤ 480 MeV was found. The account for the poles and zeros of S-matrix strongly violates the smoothness of $S(L)$. Due to that the character of refraction and absorption at $L \leq L_{sa}$ in this approach substantially differs from that of the discussed above (see curves 1, 2 in Fig. 1, *a, c*).

4. GENETIC ALGORITHM BASED NUMERICAL DIAGONALIZATION OF HERMITIAN MATRIX. APPLICATION TO NILSSON MODEL

The variability of shapes of atomic nuclei is an important property of nuclear matter. It is well established that the majority of nuclei are deformed both in ground and excited states. The study of the influence of the nuclear deformations in different states on various observables are permanently in the focus [26].

One of the most simple but physically rich approaches to the systematics of the energy level bands of the deformed nuclei is Nilsson's [27] model in which the nucleons are placed in the self-consistent field described by the oscillator potential with axial symmetry.

The key point in the calculations of the one-particle states in this model is the diagonalization of the Hamiltonian matrix in the basis of the one-particle wave functions of the spherical oscillator potential.

The full solution of the problem of searching eigenfunctions and eigenvalues of the Hermitian matrix can be found using the most developed and effective Jacobi's method – the so-called plane rotations one. The method consists in the successive transformation of the initial matrix $H = H_0$ into the more simple form by multiplying it with the matrix of plane rotation U. As a result there arises a series H_0, H_1, H_2,… in which $H_k = U_k H_{k-1} U_k^{-1}$ and U_k is selected to make the matrix H_k not contain the non-diagonal element of the matrix H_{k-1}, having the maximal absolute value. If $H_k = \left\| h_{ij}^{(k)} \right\|$, $\left| h_{pq}^{(k-1)} \right| = \max_{i \neq j} \left| h_{ij}^{(k-1)} \right|$ $U_k = \left\| u_{ij}^{(k)} \right\|$, then for the real matrix H we have

$$u_{pp}^{(k)} = u_{qq}^{(k)} = \cos\varphi\,,\ u_{pq}^{(k)} = -u_{qp}^{(k)} = \sin\varphi\,,$$

$$tg\,2\varphi = \frac{2h_{pq}^{(k-1)}}{h_{pp}^{(k-1)} - h_{qq}^{(k-1)}}\,,\ |\varphi| \le \pi/4\,. \qquad (4)$$

The sequence of the matrices H_k asymptotically squarely converges to the diagonal matrix. Besides, all the diagonal elements of the final matrix H_k turn out to be the approximate eigenvalues of H while the rows of the matrix $U^k = U_1 U_2 \ldots U_k$ become the approximate eigenvectors. Within this method the most time-consuming procedure is the search for the non-diagonal element of the diagonalized matrix, having maximal absolute value. Thus in practice various methods are employed to accelerate or optimize this process.

From the other hand, the full solution of the problem of searching eigenfunctions and eigenvalues of the Hermitian matrix is provided with help of the unitary transformation describing n - dimensional rotation (n is the dimension of the diagonalized matrix) which, using the generalized Euler angles [28], can be presented in the form:

$$g = g^{(n-1)} \ldots g^{(1)},\ g^{(k)} = g_1\left(\theta_1^k\right) \ldots g_k\left(\theta_k^k\right),$$

$$g_j\left(\theta_j^k\right) \Rightarrow \begin{cases} x_j' = x_j \cdot \cos\left(\theta_j^k\right) + x_{j+1} \cdot \sin\left(\theta_j^k\right) \\ x_{j+1}' = -x_j \cdot \sin\left(\theta_j^k\right) + x_{j+1} \cdot \cos\left(\theta_j^k\right) \end{cases},$$

$$1 \le k \le n-1,\ 1 \le j \le k\,, \qquad (5)$$

$$0 \le \theta_1^1, \theta_1^2, \ldots < 2\pi\,,\ 0 \le \theta_2^1, \theta_2^2, \ldots, \theta_3^1, \theta_3^2, \ldots < \pi\,. \qquad (6)$$

To this end the problem can be considered as the one of finding the optimal set of $n(n-1)/2$ values of Euler angles minimizing the sum of absolute values of all non-diagonal elements of the matrix under diagonalization in the rotated basis. To solve the problem we have applied the genetic algorithm which varies the optimized parameters independently and in parallel and has the exponential convergence rate.

The scheme of application of a genetic algorithm to the problem under study is as follows. Each of the optimized Euler angles is presented via the binary encoding. The population consists of the fixed number of sets of these angles. Initially all the angles are set randomly in the specified intervals (6). Then for each set of angles the unitary transformation matrix (5) is calculated and applied to the diagonalized matrix and the sum of absolute values of all non-diagonal elements of the transformed matrix is determined. Further two parent sets of angles are selected from the population according to the prescription – the less the sum of absolute values of non-diagonal elements the greater the probability to be selected. The selected sets of angles are replicated to produce offsprings. During the replication, some bits of the binary codes representing the angles are flipped with some probability (mutation). After that, the exchange of the complementary portions of bits between the parent sets of angles takes place with some probability (crossover). For each offspring the sum of the absolute values of all non-diagonal elements of the transformed primary matrix is calculated. If the value of offspring's sum becomes less the largest value of individual's sum in the population, the latter is substituted by the former. The same way the next pair of parents is selected and the process proceeds until the required quality of diagonalization is achieved. Then the diagonal elements of the matrix $H' = gHg^{-1}$ are the approximate eigenvalues of H and the rows of the matrix g are the approximate eigenvectors.

Using the dianalization method presented above, we have independently replicated the one-particle energy states scheme found by Nilsson via Jacobi's method. The comparative analysis of the developed approach and the conventional ones (e.g., Jacobi's) shows that the higher the dimension of the matrix under diagonalization the higher the efficacy of the genetic approach.

5. CONCLUSION

The proposed model-free approach allows to extract the numerical dependencies of the modulus of S–matrix and the nuclear phase on the angular momentum directly from the measured differential cross sections of elastic scattering. This approach is free from the *a priori* visions of the shape of the nuclear part of scattering matrix. The results of analyses of the differential cross sections of elastic ^{16}O–^{16}O-scattering at 22…44 MeV/nucleon featured by the pronounces refractive picture of rainbow scattering witness that the quantitative description of the available experimental data is achieved with the use of the scattering matrix determined by the modulus and the nuclear phase being smooth monotonic functions of angular momentum while the deflection function has the form typical of the case of nuclear rainbow.

It should be noted that we have studied the case of the elastic scattering of identical nuclei limited in angular distributions by $\theta < 90°$ so that one can miss some important details of rainbow structures. From this point of view it seems reasonable to extract numerical de-

pendencies of the S-matrix modulus and nuclear phase shift on angular momentum from the differential cross sections for elastic $^{16}O–^{12}C$-scattering at $E(^{16}O) = 230…281$ MeV with help of the fitting procedure based on the evolutionary algorithm and find out whether these characteristics are smooth monotonic functions in the angular momentum space.

Nilsson's model allows successful description of the ground state properties of light nuclei with $4<A<32$, including spins, parities, magnetic and quadrupole moments, etc [29]. With help of the modified Nilsson's model [30-32] which account for the different deformations of a nucleus in different excited one-particle states the experimentally measured probabilities of electromagnetic transitions between the excited one-particle states of light nuclei have been more adequately explained. Within the specified model it is actual to recalculate the energies and the quantum characteristics of the excited one-particle states of light nuclei, being the ones equilibrium in deformation, and find out whether it is possible to agree the calculated and the measured data if we take into account the dependencies on nuclear deformation of the weights with which the operators of the spin-orbit interaction and the interaction proportional to the square of angular momentum enter the Hamiltonian.

This study was partly supported by The State Fund of Fundamental Research of the Ukraine (grant 02.07/372).

REFERENCES

1. M.E. Brandan, G.R. Satchler. The interaction between light heavy-ions and what it tells us // *Phys. Rep*. 1997, v. 285, №4, 5, p. 143-243.

2. W. von Oertzen, A. Blazevich, H.G. Bohlen et al. . Nuclear rainbow scattering and nucleus-nucleus potentials at small distances // *Yad. Fiz.* 2002, v. 65, №4, p. 710-714.

3. K.W. McVoy, M.E. Brandan. The 90° excitation function for elastic $^{12}C + ^{12}C$ scattering. The importance of Airy elephants // *Nucl. Phys. A.* 1992, v. 542, №2, p. 295-309.

4. S.G. Cooper, R.S. Mackintosh. $^{11}Li + ^{28}Si$ and $^{11}Li + ^{12}C$ elastic scattering studied by inversion // *Nucl. Phys. A.* 1995, v. 582, №1, p. 283-295.

5. S.G. Cooper, M.A. McEvan, R.S. Mackintosh. Elastic scattering phenomenology by inversion: ^{16}O on ^{12}C at 608 MeV // *Phys. Rev. C.* 1992, v. 45, №2, p. 770-773.

6. S.G. Cooper, R.S. Mackintosh. $^{16}O+^{16}O$ elastic scattering at 350 MeV studied by inversion // *Nucl. Phys. A.* 1994, v. 576, №2, p. 308-316.

7. A.M. Kobos, M.E. Brandan, G.R. Satchler. Further optical model studies of ^{16}O scattering at *E/A* = 94 MeV // *Nucl. Phys. A.* 1988, v. 487, №3, p. 457-476.

8. Yu.A. Berezhnoy, V.V. Pilipenko. Analysis of refraction effects in nuclear scattering on the basis of the S-matrix approach // *Mod. Phys. Lett. A.* 1995, v. 10, №31, p. 2305-2312.

9. D. Diver. Application of genetic algorithms to the solution of ordinary differential equations // *J. of Phys. A: Math. and Gen.* 1993, v. 26, p. 3503- 3513.

10. J.R. Morris, D.M. Deaven, K.M. Ho. Genetic-algorithm energy minimization for point charges on a sphere // *Phys. Rev. B.* 1996, v. 53, №4, p. R1740-R1743.

11. S.V. Berezovsky, V.Yu. Korda, V.F. Klepikov. Multi-level genetic-algorithm optimization of the thermodynamic analysis of incommensurate phase in ferroelectric $Sn_2P_2Se_6$ // *Phys. Rev. B.* 2001, v. 64, №6, p. 3.1-3.7.

12. C. Winkler, H.M. Hofmann. . Determination of bound-state wave functions by a genetic algorithm // *Phys. Rev. C.* 1997, v. 55, №2, p. 684-687.

13. C. Winkler, H.M. Hofmann. Structure of the helium isotopes ^4He-^8He // *Phys. Rev. C.* 1997, v. 55, №2, p. 688-698.

14. K. Michaelian. Evolving and energy dependent optical model description of heavy-ion elastic scattering // *Revista Mexicana de Fisica.* 1996, v. 42 (suppl. 1), p. 203-215.

15. V.Yu. Korda. Implementation of a genetic algorithm for analyzing spectra of quantum systems // *J. of Kharkiv Nat. Univ., Phys. Series: Nuclei, Particles, Fields.* 2001, №510, issue 1 (13), p. 41-43.

16. D.G. Ireland. Using a genetic algorithm to investigate two-nucleon knockout reactions // *J. of Phys. G: Nuclear and Particle Physics.* 2000, v. 26, p. 157-166.

17. W.E. Frahn. *Diffractive Processes in Nuclear Physics.* Oxford: "Clarendon Press", 1985, 179 p.

18. V.Yu. Korda, A.S. Molev, L.P. Korda. Evolving model-free scattering matrix via evolutionary algorithm // *J. of Kharkiv Nat. Univ., Phys. Series: Nuclei, Particles.* 2002, №559, issue 2 (18), p. 37-39.

19. Dao T. Khoa, W. von Oertzen, H.G. Bohlen et al. // Study of diffractive and refractive structure in the elastic $^{16}O + ^{16}O$ scattering at incident energies ranging from 124 to 1120 MeV // *Nucl. Phys. A.* 2000, v. 672, №2, p. 387-416.

20. A.V. Kuznichenko, V.V. Pilipenko, G.M. Onishchenko, A.S. Dem'yanova, N. Burtebaev. Refractive effects in $^{16}O+^{16}O$ scattering at 124-1120 MeV and the S-matrix model with Regge poles // *Izvestiya RAN, Physics Series.* 2003, v. 67, №5, p. 704-709. (in Russian).

21. E. Stiliaris, H.G. Bohlen, P. Fröbrich et al. Nuclear rainbow structures in the elastic scattering of ^{16}O on ^{16}O at E_L = 350 MeV // *Phys. Lett. B.* 1989, v. 223, №3, p. 291-295.

22. M.E. Brandan, G.R. Satchler. Optical potential ambiguities and $^{16}O + ^{16}O$ at 350 MeV // *Phys. Lett. B.* 1991, v. 256, №3, 4, p. 311-315.

23. M.E. Brandan, G.R. Satchler. Folding model analysis of $^{12, 13}C + ^{12}C$ and $^{16}O + ^{12}C$ scattering at intermediate energies using a density-dependent interaction // *Nucl. Phys. A.* 1988, v. 487, №3, p. 477-492.

24. A.S. Molev, V.Yu. Korda. *Analysis of refractive effects in $^{16}O – ^{16}O$-scattering on the basis of S–matrix*

approach with the use of a genetic algorithm. Proc of the 52th Meeting on nuclear spectroscopy and nuclear structure (NUCLEUS-2002), 2002, p. 164. (in Russian).

25. A.S. Molev. Extraction of refractive and diffractive contributions to cross sections for inelastic scattering of light ions with E ≥ 20 MeV/nucleon by nuclei // *Izvestiya RAN, Physics Series.* 1995, v 59, №1, p. 96-102. (in Russian).

26. H. Röpke, P.M. Endt. Renaissance of the Nilsson-model approach to light nuclei: The case of the A=26 system. // *Nucl. Phys. A.* 1998, 632, №2, p. 173-204.

27. S.G. Nilsson. Bound states of individual nucleons in highly deformed nuclei //*Kgl. Danske Videnskab. Selkab. Mat.-fys. Medd.* 1955, v. 29, №16, p. 1-68.

28. N.Ya. Vilenkin. Special functions and the theory of representations of groups. Moscow: "Nauka", 1991, 576 p. (in Russian)

29. V.Yu. Gonchar, E.V. Inopin, S.P. Tsytko. *Light nuclei and generalized model.* Preprint KIPT, 1959, Д-001, 37 p. (in Russian)

30. E.V. Inopin, E.G. Kopanets, L.P. Korda, V.Ya. Kostin, A.A. Koval'. Electromagnetic transitions in nuclei between states with different deformations // *Problems of Atomic Science and Technology. Series: Nuclear Physics Investigations.* 1975, №3(15), p. 31-33. (in Russian)

31. E.G. Kopanets, E.V. Inopin, L.P. Korda. Analog-antianalog M1-transition at different deformations of initial and final states // *Izvestiya AN SSSR, Physics Series*, 1975, v. 10, №39, p. 2032-2033. (in Russian)

32. E.G. Kopanets, E.V. Inopin, L.P. Korda, V.Ya. Kostin, A.A. Koval'. *M*1 and *E*2 transitions between nuclear states with different deformations // *Izvestiya AN SSSR, Physics Series*, 1976, v. 4, №40, p. 780-783. (in Russian)

ПРИМЕНЕНИЕ ЭВОЛЮЦИОННЫХ АЛГОРИТМОВ ДЛЯ БЕЗМОДЕЛЬНОГО ОПРЕДЕЛЕНИЯ МАТРИЦЫ РАССЕЯНИЯ И ЧИСЛЕННОЙ ДИАГОНАЛИЗАЦИИ ГАМИЛЬТОНИАНА В МОДЕЛИ НИЛЬСОНА

А.Н. Водин, В.Ю. Корда, А.Н. Довбня, А.С. Молев, Л.П. Корда

Предложен новый безмодельный подход для анализа дифференциальных сечений упругого рассеяния легких ядер промежуточных энергий ядрами, позволяющий извлекать численные зависимости модуля и ядерной фазы матрицы рассеяния от орбитального момента непосредственно из экспериментальных данных с помощью эволюционного алгоритма. Изучены преломляющие и поглощающие свойства ^{16}O–^{16}O-взаимодействия при E = 22…44 МэВ/нуклон. Представлен новый метод диагонализации матрицы гамильтониана в модели Нильссона деформированного ядра с аксиальной симметрией, использующий генетический алгоритм. Тестирование показало, что чем больше размер диагонализуемой матрицы, тем эффективнее оказывается генетический подход по сравнению с традиционными методами диагонализации.

ЗАСТОСУВАННЯ ЕВОЛЮЦІЙНИХ АЛГОРИТМІВ ДЛЯ БЕЗМОДЕЛЬНОГО ВИЗНАЧЕННЯ МАТРИЦІ РОЗСІЯННЯ І ЧИСЕЛЬНОЇ ДІАГОНАЛІЗАЦІЇ МАТРИЦІ ГАМІЛЬТОНІАНУ В МОДЕЛІ НІЛЬСОНА

О.М. Водін, В.Ю. Корда, А.М. Довбня, О.С. Молєв, Л.П. Корда

Запропоновано новий безмодельний підхід для аналізу диференціальних перерізів пружного розсіяння легких ядер проміжних енергій ядрами, що дає змогу визначати числові залежності модуля і ядерної фази матриці розсіяння від орбітального моменту безпосередньо з експериментальних даних за допомогою еволюційного алгоритму. Досліджені заломлюючі та поглинаючі властивості ^{16}O–^{16}O-взаємодії при E = 22…44 МеВ/нуклон. Представлено новий метод діагоналізації матриці гамільтоніану в моделі Нільсона деформованого ядра з аксіальною симетрією, що застосовує генетичний алгоритм. Тестування довело, що чим більше розмір діагоналізуємої матриці, тим ефективнішим виявляється генетичний підхід порівняно з традиційними методами діагоналізації.